

BATU-EXAM

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Engg Mechanics Department

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Mechanics & Electrical & Electronics

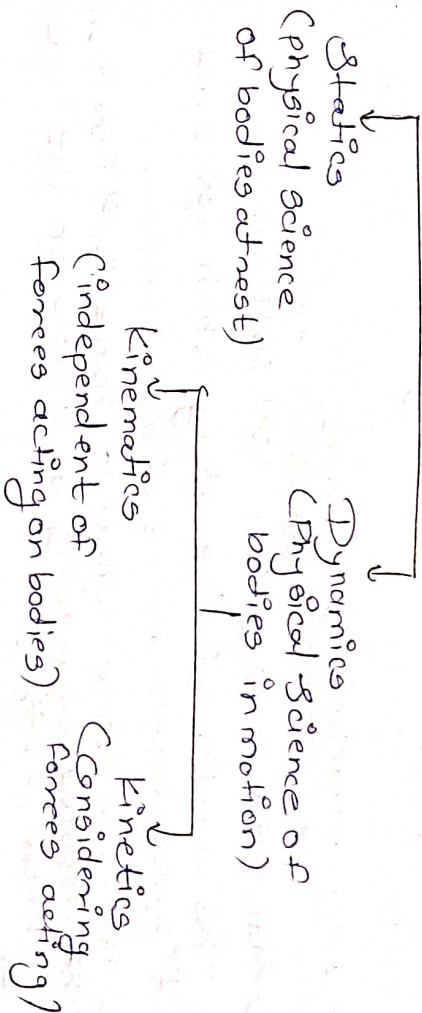
Mechanics

20/08/23

Definitions-

It is the branch of applied science which deals with laws and principles of mechanics along with their application to engineering problem.

Divisions of Engineering Mechanics.



* Statics:

- It is the branch of eng. mechanics which deals with forces & their effect while acting upon at rest.

* Dynamics:

- It is the branch of eng. mechanics which deals with forces & their effect while acting upon the bodies in motion. It is further divided into Kinematics and Kinetics.

① Kinematics:

- It is the branch of dynamics which deals with bodies in motion without any reference to forces which are responsible for motion.
- eg. Projectile.

Kinetics:

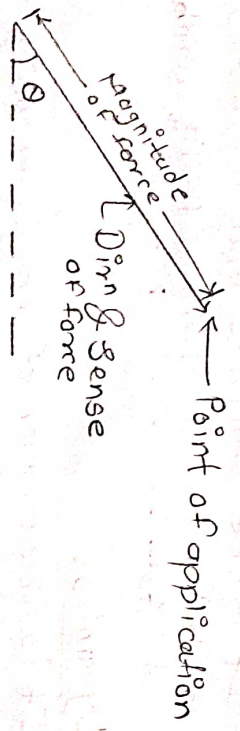
It is the branch of dynamics which deals with bodies in motion due to application of forces.

Forces:

The force is defined as an agent which produces or tends to produce, destroy or tend to destroy motion.

Characteristics of forces:

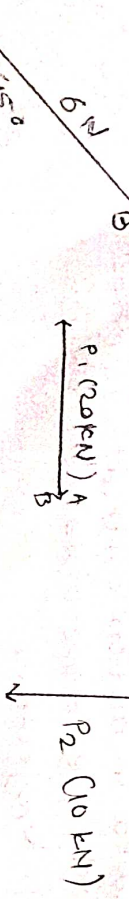
- ① Magnitude of force (i.e. 100N, 50N, etc)
- ② The direction of line along which force act. It is also known as line of action of force.
- ③ Nature of force (whether force is Pull or Push). This denoted by placing an arrow on the line of action of forces.
- ④ The point at which the force act on the body.



Representation of forces.

Forces may be represented by methods

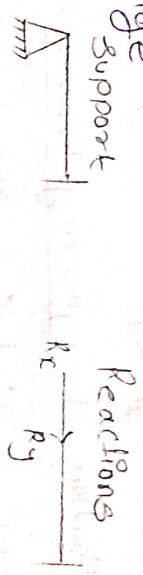
① Vector representation - This method represents force graphically by vector.



② Bow's Notation - It is method of representing a force by writing two Capital letters on either side of the force.

Types of Support:

① Pin or Hinge Support



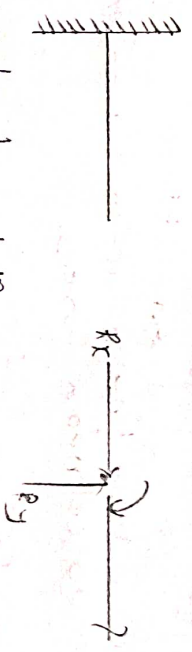
It prevents translation along x & y.

② Roller -



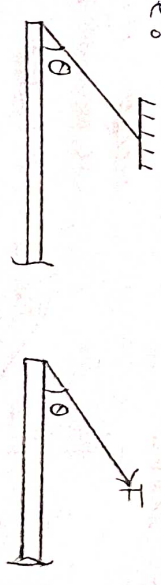
It prevents translation along the direction perpendicular to the roller surface.

③ Fixed:



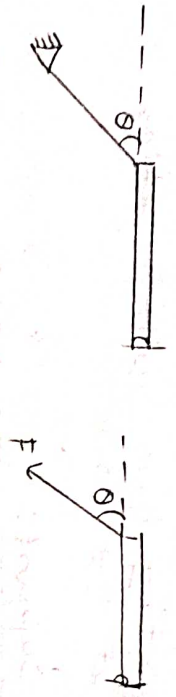
It prevents translation along x and y direction. It is also prevents rotation of body.

④ Cable:



Tension force acts along direction of cable away from body.

5) Link.



Reaction acts along the link away towards the body.

Idealisation Of Engineering Problems:

- 1) Strategy
- 2) Modeling
- 3) Analysis
- 4) Reflect & Think.

A system of forces - when two or more forces acts on a body they are called as system of forces.

1) Coplanar forces:

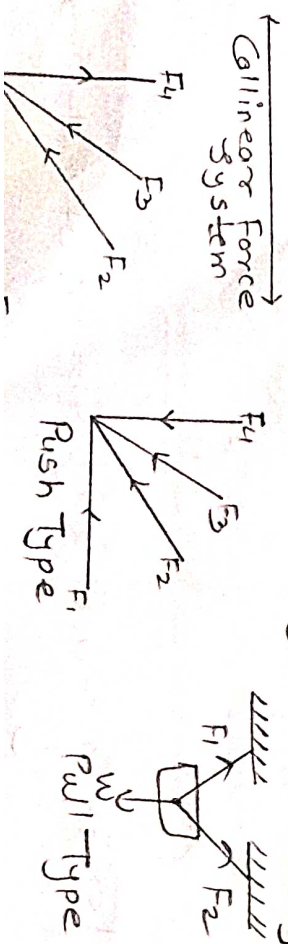
The forces whose line of action lie on some plane are known as coplanar forces.

2) Collinear forces:

The force whose line of action lie on same line are known as collinear forces.

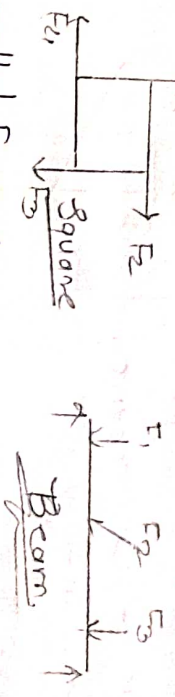
3) Concurrent forces:

The force which meet at one single point is known as concurrent forces



4) Non-Concurrent forces:

The forces which acts at different point are known as non-concurrent forces.



5) Parallel Forces:

The forces whose line of action are parallel to each other are called as parallel forces.

They may be classified as:

1) Like parallel forces: Parallel forces acting in same direction are called like parallel forces.

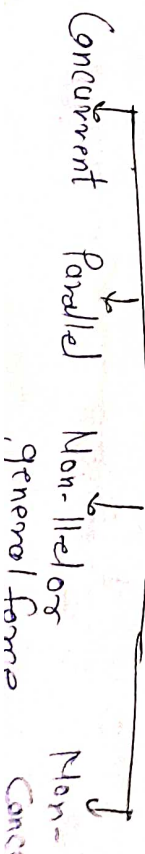
2) Unlike parallel forces: Parallel force acting in opposite direction are called as unlike parallel forces.



6) Non-Coplanar Force System:

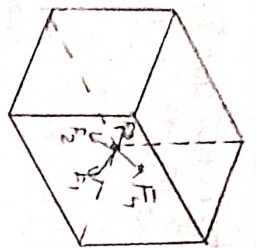
If line of action of all forces lie on different plane the system is called as non-coplanar force system.

Non-Coplanar force system.



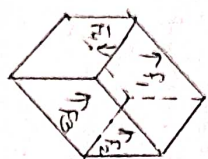
1) Non-Coplanar Concurrent:

The forces lie in different plane. But passes through single point of concurrency 'O' as shown in figure.



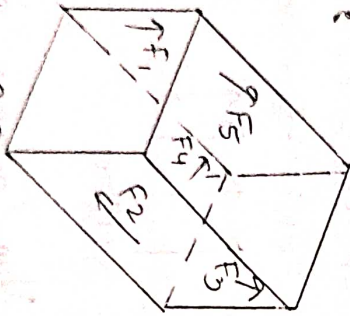
2) Non-Coplanar parallel:

This forces exist in different plane but parallel to each other.



3) Non-Coplanar Non-Concurrent:

This forces act in different plane and they do not pass through one single point of concurrency as shown in figure.

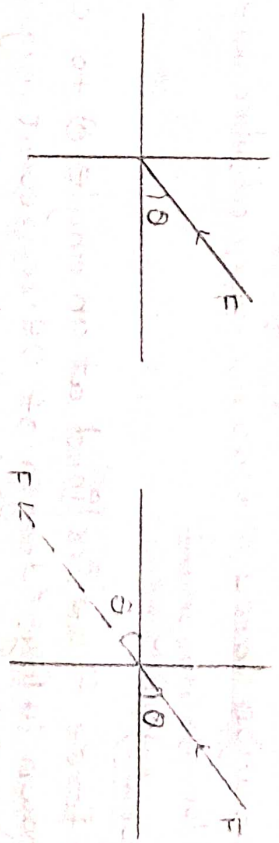


Principle of Transmissibility of forces:-

It states that "If a force acts at any point on a rigid body it may also be considered to act at any other point on its line of action, provided this point is rigidly connected with body."



From Fig F is the force act at a point A, in the rigid body. The same force will act at any other point on the line BC without change in its magnitude.



As per this principle if the force is pushed in any quadrant the component can be calculated by extending the line of action of force and making it pull.

Resultant \rightarrow Sum of all the forces acts the body

* Resultant forces:-

- If a number of forces are acting at simultaneous on a point or particle then it is possible to find out a single force which could replace them i.e, which could produce the same effect as produced by given forces. This single force is called as resultant force & the given forces are called component of forces.

* Resolution of Force -

- The way of representing a single force into number of forces, without enhancing the effect of the force on the body is called resolution of forces.

Methods of Resolution - There are two methods of resolution,

① Resolution of two forces X two mutually perpendicular component (Orthogonal)

② Resolution Force X two non-perpendicular component (Non-Orthogonal)

(Orthogonal)

① Let a force F be inclined at an angle θ to x -axis as shown in Fig. (Length of OA represent magnitude of F). we have to resolve it into comp. two components F_x along x -axis & F_y along y -axis.

Given: F & θ

To find: F_x & F_y

In ΔOAB , $\cos\theta = \frac{OB}{OA}$

$\therefore OB = F_x = F \cos\theta$

$\& \sin\theta = \frac{AB}{OA}$

$\therefore AB = F_y = F \sin\theta$

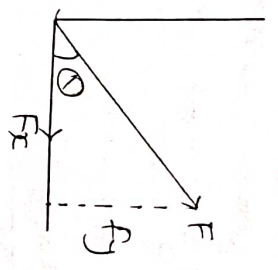
Magnitude of components $F_x = F \cos\theta$ & $F_y = F \sin\theta$
 Direction of components is F_x towards right & F_y vertically upward.

Different cases of resolution of force.

- , +	+ , +
- , -	+ , -

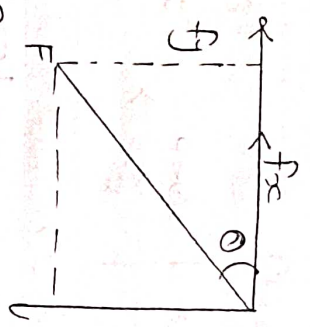
Part (I) : Resolution when the force is pull in respective quadrant.

Case (I)



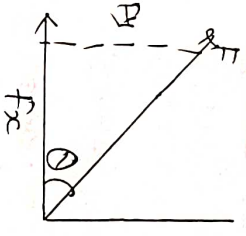
$F_x = F \cos\theta$
 $F_y = F \sin\theta$

Case (II)



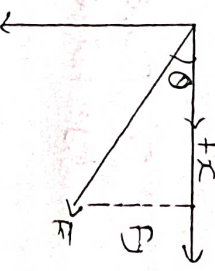
$F_y = -F \sin\theta$
 $F_x = -F \cos\theta$

Case (III)



$F_x = -F \cos\theta$
 $F_y = F \sin\theta$

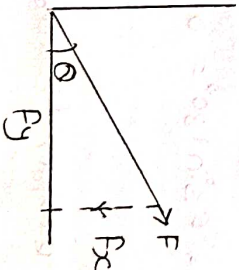
Case (IV)



$F_x = F \cos\theta$
 $F_y = -F \sin\theta$

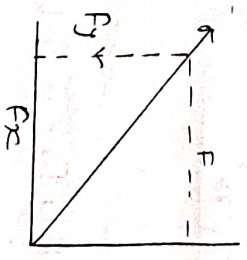
Part (II) - Resolution when force is push in respective quadrant.

Case (I)



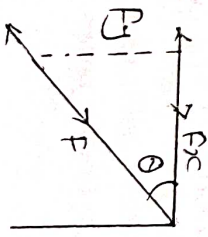
$F_x = F \cos\theta$
 $F_y = F \sin\theta$

Case (II)



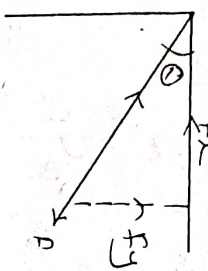
$F_x = F \cos\theta$
 $F_y = -F \sin\theta$

Case (III)



$F_x = F \cos\theta$
 $F_y = F \sin\theta$

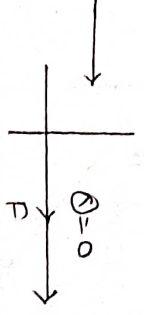
Case (IV)



$F_x = -F \cos\theta$
 $F_y = F \sin\theta$

Part (III): Resolution when force lies on x and y axis.

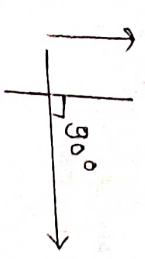
Case (I)



$$F_x = F \cos 0 = F$$

$$F_y = F \sin 0 = F \sin 0 = 0$$

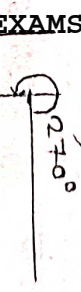
Case (II)



$$F_x = F \cos 90^\circ = 0$$

$$F_y = F \sin 90^\circ = F$$

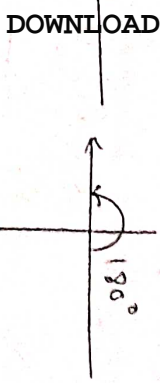
Case (III)



$$F_x = F \cos 180^\circ = -F$$

$$F_y = F \sin 180^\circ = 0$$

Case (IV)

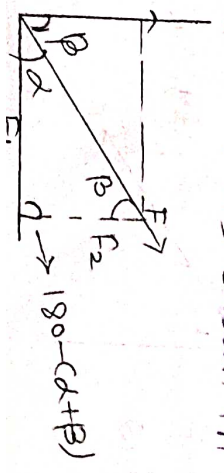


$$F_x = F \cos 270^\circ = 0$$

$$F_y = F \sin 270^\circ = -F$$

Resolution of a force into two Non-perpendicular Component (Non-Orthogonal) -

Let F_1 and F_2 be component of F along axis α and β with F as shown in Fig.



$$F_n \triangle OAB$$

$$\frac{F_1}{\sin \beta} = \frac{F_2}{\sin \alpha} = \frac{F}{\sin (180^\circ - (\alpha + \beta))}$$

$$F_1 = \frac{F \sin \beta}{\sin (180^\circ - (\alpha + \beta))}$$

$$F_2 = \frac{F \sin \alpha}{\sin (180^\circ - (\alpha + \beta))}$$

Composition of Forces -

The process of finding out resultant force of numbers of given forces is called composition of forces. The resultant of a force system can be determined by any one of the following methods:-

- (I) Analytical Method
- (II) Graphical.

(I) Analytical Method:

(A) Resultant of Coplanar, collinear Forces:-

Let us consider collinear forces F_1, F_2, F_3 , as shown in Fig for finding their resultant analytically, algebraic sum of their sense & get net value which will give the resultant.

The direction of resultant depends upon their magnitude.



(B) Resultant of coplanar, concurrent forces:-

(I) Method of Resolution:-

For (I) Resolve all the forces horizontally & vertically and find algebraic sum of all horizontally and vertically component i.e. ΣF_x & ΣF_y .

The resultant (R) will be given by

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$$

Let θ be acute angle made by resultant with horizontal then $\tan \theta = \frac{\sum F_y}{\sum F_x}$

$$\theta = \tan^{-1} \left(\frac{\sum F_y}{\sum F_x} \right) \text{ --- direction}$$

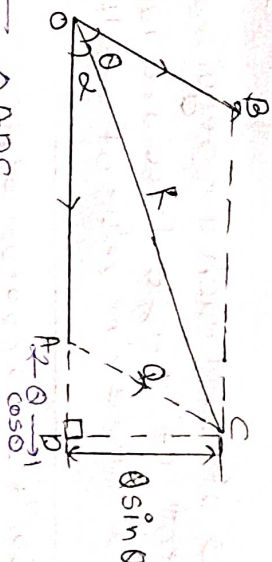
$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2} \text{ --- magnitude}$$

Parallellogram law of Forces:-

Parallellogram law of forces state that "if two forces acting simultaneously on a particle be represented in magnitude and direction by two adjacent sides of Parallellogram then resultant may be represented in magnitude and direction by the diagonal of parallellogram which passes through their point of intersection."

Theorem:

Let us consider two concurrent forces P and Q acting at and away from origin O. Let this two forces be represented in magnitude and direction as two adjacent sides of parallellogram OACB. Thus line joining O & C represent resultant (R) in magnitude & direction according to parallellogram law of forces.



In ΔADC

$$\sin \theta = \frac{CD}{AC} \quad \cos \theta = \frac{AD}{AC}$$

$$CD = P \sin \theta \quad AD = P \cos \theta$$

In ΔOCD

$$OC^2 = OD^2 + CD^2$$

$$R^2 = (P + Q \cos \theta)^2 + (Q \sin \theta)^2$$

$$= P^2 + 2PQ \cos \theta + Q^2 (\cos^2 \theta + \sin^2 \theta)$$

$$= P^2 + 2PQ \cos \theta + Q^2 (\cos^2 \theta + \sin^2 \theta)$$

$$R^2 = P^2 + Q^2 + 2PQ \cos \theta$$

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

Magnitude.

$$\tan \alpha = \frac{CD}{OD} = \frac{Q \sin \theta}{P + Q \cos \theta}$$

$$\alpha = \tan^{-1} \left(\frac{Q \sin \theta}{P + Q \cos \theta} \right)$$

Direction.

* Resultant of coplanar Non-concurrent forces-

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$$

$$\theta = \tan^{-1} \left(\frac{\sum F_y}{\sum F_x} \right)$$

* Ques: If two forces of 60 N each are required to be equivalent of a single force are having angle 120°. Calculate the value of resultant.

Soln:-
 $R = \sqrt{(60)^2 + (60)^2 + 2(60 \times 60 \cos 120^\circ)}$

$$= \sqrt{3600 + 3600 + 2(3600 (-\frac{1}{2}))}$$

$$= \sqrt{3600 + 3600 - 3600}$$

$$= \sqrt{3600}$$

$$R = 60$$

$$\tan \alpha = \frac{60 \sin 120^\circ}{60 + 60 \cos 120^\circ}$$

$$\alpha = \tan^{-1}(1.73)$$

= 60° ————— with 60 N force.

2) Find angle betn two equal forces P if their resultant is equal to P/2.

Soln:- $P = P, Q = P, R = \frac{P}{2}, Q = ?$

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$\frac{P}{2} = \sqrt{P^2 + P^2 + 2P^2 \cos \theta}$$

$$\frac{P}{2} = \sqrt{2P^2 (1 + \cos \theta)}$$

$$\frac{P}{2} = \sqrt{2P^2 \cdot 2 \cos^2 \frac{\theta}{2}}$$

$$\frac{P}{2} = 4P^2 \cos^2 \left(\frac{\theta}{2}\right)$$

$$\cos^2 \frac{\theta}{2} = \frac{1}{16}$$

$$\cos \frac{\theta}{2} = \frac{1}{4}$$

$$\frac{\theta}{2} = \cos^{-1}\left(\frac{1}{4}\right)$$

$$\frac{\theta}{2} = 75.5^\circ$$

$$\theta = 75.5^\circ \times 2$$

$$\theta = 151^\circ$$

(Q3) Determine the angle betn two forces & resultant & one of the force.

Soln:- Given, $P = 12 \text{ N}, Q = 9 \text{ N}, R = 15 \text{ N}.$

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$15 = \sqrt{(12)^2 + (9)^2 + 2 \times 12 \times 9 \times \cos \theta}$$

$$15 = \sqrt{144 + 81 + 216 \cdot \cos \theta}$$

$$15 = \sqrt{225 + 216 \cdot \cos \theta}$$

$$225 = 225 + 216 \cdot \cos \theta.$$

$$= 216 \cdot \cos \theta = 0$$

$$\therefore \cos \theta = 0$$

$$\therefore \theta = \cos^{-1}(0)$$

$$\therefore \theta = 90^\circ \text{ or } \theta = 270^\circ$$

$$\tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$$

$$= \frac{9 \sin 90}{12 + 9 \cos 90^\circ}$$

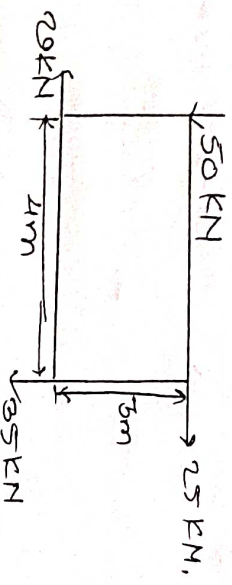
$$= \frac{9}{3}$$

$$\therefore \alpha = \tan^{-1}\left(\frac{3}{1}\right)$$

$$= 36.86^\circ \text{ with } 12 \text{ N force.}$$

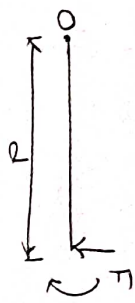
Ques. A system of forces are acting at corner of rectangular block as shown in fig. determine magnitude & direction of resultant force.

Soln:-



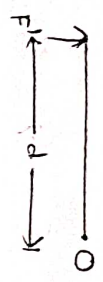
Q- Classification of moment according to direction of rotation:-

① Clockwise Moment.



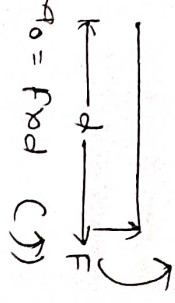
Moment of F about O

$$M_o = F \times d \quad (2)$$

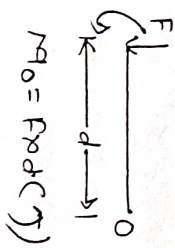


$$M_o = F \times d \quad (2)$$

② Anticlockwise Moment



$$M_o = F \times d \quad (2)$$



$$M_o = F \times d \quad (2)$$

Sign Conventions

Clockwise Moment \rightarrow +ve

Anticlockwise Moment \rightarrow -ve

Law of moment:-

It states that if a body is in rotational equilibrium under the action of numbers of forces the sum of clockwise moment is equal to sum of anticlockwise moment of forces about the same point.

* Varignon's Theorem:-

It is states the algebraic sum of all forces about any point is equal to resultant is equal to moment of resultant about the same point.

$$\sum M_A = M_{RA}$$

i.e, $F_1 r_1 + F_2 r_2 + F_3 r_3 + \dots + F_n r_n = R r$

where

$\sum M_{FA}$ = Algebraic sum of moment of forces about point A

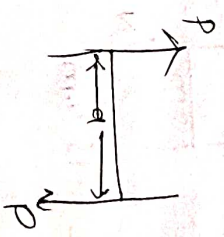
M_{RA} = Moment of resultant about point A

* Couple:-

- Two equal unlike, parallel, non-collinear force, form a Couple. As the forces are equal & opposite their resultant is zero hence couple only produces pure rotary motion without producing linear motion.

* Lever Arm:-

The dist. betn two forces of couple is known as lever arm. (unit: Nm)



a = lever arm or arm of Couple

Properties of Couple;

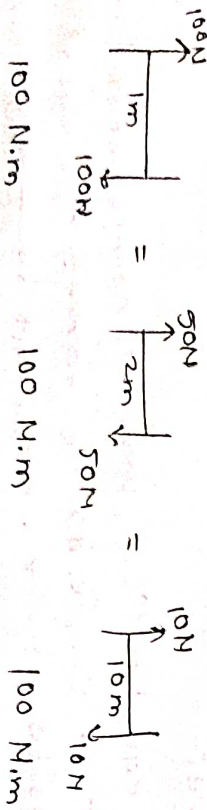
① Resultant of forces of couple is zero.
($R = P - P = 0$)

② The moment of couple is equal to the product of one of forces and arm of the couple
 $M = P \times d$

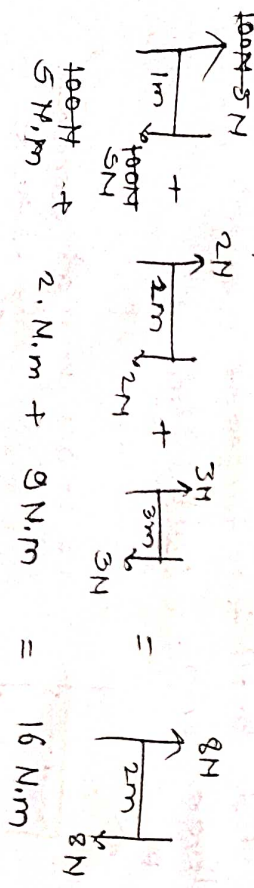
③ Moment of couple at any point is constant.

④ A couple can be balance by another couple of equal and opposite moment.

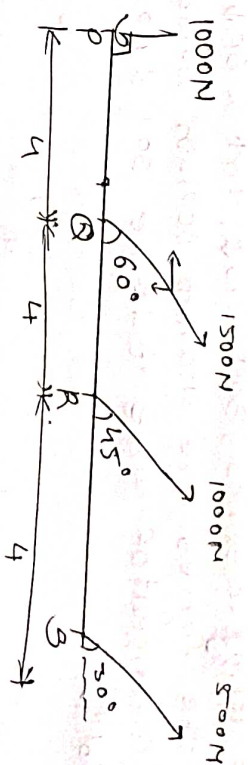
⑤ Two or more are said to be equal when they have same sense.



Any number of coplanar couple can be presented by a single couple. The moment of which is equal to algebraic sum of moments of all couple.



Ex. A horizontal line PQRS is 12m long. Force $PQ = QR = RS = 4m$ process of 1000 N, 1500 N, 1000 N, & 500 N at P, Q, R, S resp. with downward direction. The line of action of the process make angle of $90^\circ, 60^\circ, 45^\circ, \& 30^\circ$ resp. with PS. Find magnitude direction and position of resultant force.



$$\sum F_x = -1000 \cos 90^\circ - 1500 \cos 60^\circ - 1000 \cos 45^\circ - 500 \cos 30^\circ = -1890.11 \text{ N}$$

$$\sum F_y = -1000 \sin 90^\circ - 1500 \sin 60^\circ - 1000 \sin 45^\circ - 500 \sin 30^\circ = -3256.14 \text{ N}$$

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$$

$$= 3764.96 \text{ N}$$

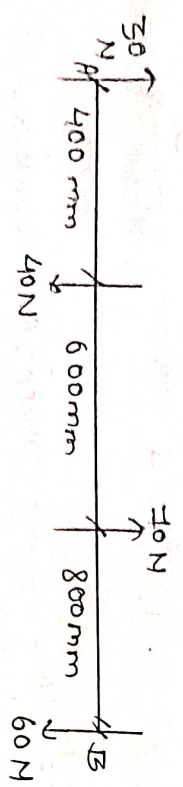
$$\theta = \tan^{-1} \left(\frac{\sum F_y}{\sum F_x} \right)$$

$$= 59.8^\circ$$

By applying Varignon's theorem

$$\sum M_{EP} = M_{RP}$$

Problem: Four forces of 30 N, 70 N acting upward, one balance by force 60 N & 40 N acting downward. Force acting in series 300, 400, 700 & 600 dist betn the forces are 400 mm, 600 mm, 800 mm resp. Find movement of couple of force 30 N & 60 N.



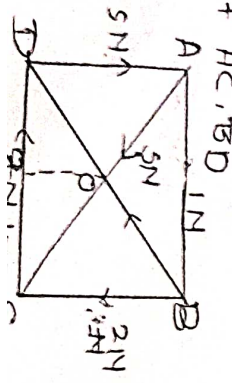
$$M_A = +40 \times 400 - 70 \times (400 + 600) + 60 \times (400 + 600 + 800)$$

$$= \boxed{54000 \text{ N}\cdot\text{mm} \text{ (2)}}$$

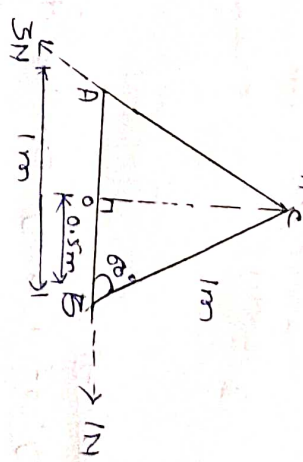
$$M_B = 70 \times 800 - 40 \times (800 + 600) + 30 \times (800 + 600 + 400)$$

$$= 54000 \text{ N}\cdot\text{mm} \text{ (2)}$$

ABCD is square of 2 meters side. Forces having magnitude 1 N, 2 N, 3 N, 5 N, one acting along the side of AB, BC, CD, DA resp. In addn this force 5 N, 2 N, are acting along AC, DB resp. Calcn the moment at the point of intersection of AC, BD.



Q. Three forces 1 N, 2 N, 3 N, are acting the side of equilateral triangle of side 1 m along AB, BC, & CA. Find algebraic sum of moment of all forces about point C.



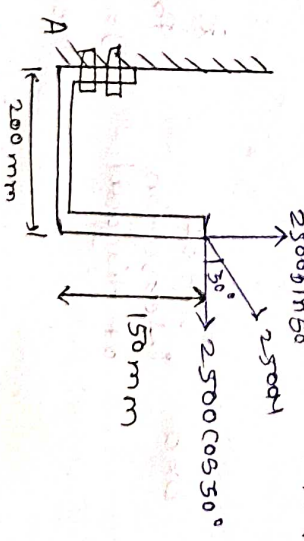
$$M_C = -1 \times 0.866$$

$$= -0.866 \text{ Nm}$$

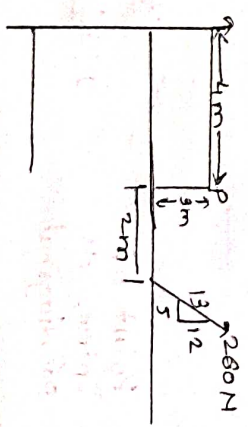
$$= 0.866 \text{ Nm (5)}$$

$$OC = 0.866 \text{ m}$$

Q. A force 2500 N acts at bracket as shown in fig. Find moment of this force at point A.



Q. Determine magnitude & directional sense of resultant moment of force about point P.



Centre of Gravity:
 Centre of gravity of a body can be define as point through which whole weight of body is assume to act. irrespective of position of body.

① Straight line
 Area -
 Position - Midpoint
 Centroid, $\bar{x} = \bar{y} = \frac{l}{2}$ from A or B

② Square
 Area $b \times b$
 Position - Point of intersection of diagonal
 Centroid, $\bar{x} = \bar{y} = \frac{b}{2}$ from AB or BC

③ Rectangle
 Area $b \times d$
 Position - Point of intersection of diagonal
 Centroid, $\bar{x} = \frac{b}{2}$ from AB, $\bar{y} = \frac{d}{2}$ from BC

④ Parallelogram
 Area $b \times d$
 Position - Point of intersection of diagonal
 Centroid, $\bar{x} = \frac{b}{2}$, $\bar{y} = \frac{d}{2}$

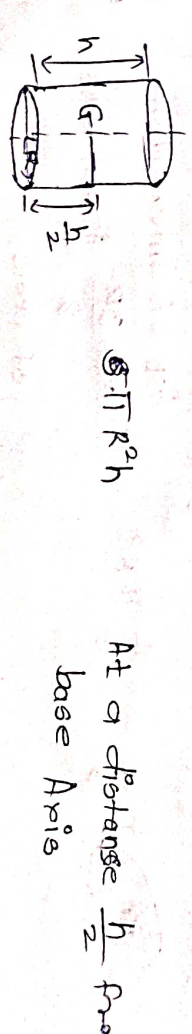
⑤ Triangle
 Area $\frac{1}{2} b \times d$
 Position - Point of intersection of medians
 Centroid, $\bar{x} = \frac{1}{3} b$ from AB, $\bar{y} = \frac{1}{3} h$ from BC

⑥ Circle
 Area πr^2 or $\frac{\pi d^2}{4}$
 Position - Centroid of circle
 Centroid, $\bar{x} = \bar{y} = \bar{r} = \frac{d}{2}$

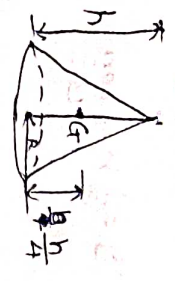
⑦ Semicircle
 Area $\frac{\pi r^2}{2}$ or $\frac{\pi d^2}{8}$
 Position - At dist. from AB
 Centroid, $\bar{x} = r$ from A, $\bar{y} = \frac{4r}{3\pi}$ from A

⑧ Quarter Circle
 Area $\frac{\pi r^2}{4}$ or $\frac{\pi d^2}{16}$
 Position - At dist. of boundary axis
 Centroid, $\bar{x} = r - \frac{4r}{3\pi}$ from A, $\bar{y} = \frac{4r}{3\pi}$ from AO

⑨ Cylinder
 Volume $\pi R^2 h$
 Position of G - At a distance $\frac{h}{2}$ from base Axis



2) Right Circular Cone



Volume

$$\frac{1}{3} \pi R^2 h$$

Position of G

At a distance $h/4$ from base axis

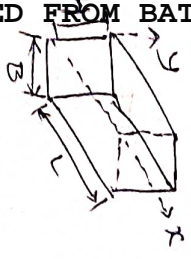
3) Sphere



$$\frac{4}{3} \pi R^3$$

At the centre of sphere

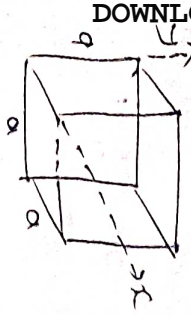
4) Solid Rectangular Block



$$L \times B \times H$$

Point of intersection of diagonals

5) Solid Cube



$$a^3$$

Point of intersection of diagonal.

6) Hemisphere



$$\frac{2}{3} \pi R^3$$

At of intersection of $\frac{3R}{8}$ from base of axis.

* Centroid for composite Plane figures:-

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3 + \dots + a_n x_n}{a_1 + a_2 + a_3 + \dots + a_n}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3 + \dots + a_n y_n}{a_1 + a_2 + a_3 + \dots + a_n}$$

* Centre of Gravity of composite Solids:-

$$\bar{x} = \frac{V_1 x_1 + V_2 x_2 + V_3 x_3 + \dots + V_n x_n}{V_1 + V_2 + \dots + V_n}$$

$$\bar{y} = \frac{V_1 y_1 + V_2 y_2 + V_3 y_3 + \dots + V_n y_n}{V_1 + V_2 + \dots + V_n}$$

where $\rightarrow V_1, V_2, V_3, \dots, V_n \rightarrow$ Volume of different Solids.

* Centroid / C.G areas/ Solids with cutout Sections-

C.G for Cutout Plane areas

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2}$$

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

C.G for cutout Solids.

$$\bar{x} = \frac{V_1 x_1 - V_2 x_2}{V_1 - V_2}$$

$$\bar{y} = \frac{V_1 y_1 - V_2 y_2}{V_1 - V_2}$$

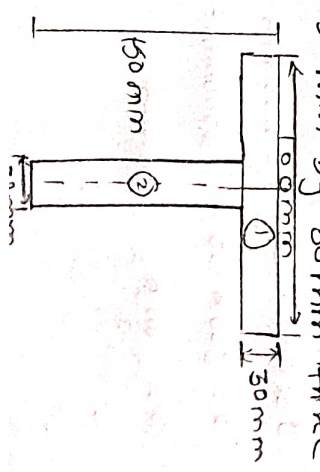
Que. Find CG of 100mm by 150mm by 20mm thick section as shown figure.

\rightarrow Solns:-

$$a_1 = 100 \times 50$$

$$= 5000 \text{ mm}^2$$

$$y_1 = \left(150 - \frac{50}{2}\right) = 135 \text{ mm}$$



(T)

15/09/23

$$A_2 = 50 \times 120 = 5600 \text{ mm}^2$$

$$y_2 = \frac{120}{2} = 60 \text{ mm}$$

$$a_1 + a_2$$

$$= 5000 \times 135 + 5600 \times 60$$

(5000 + 5600)

$$= 94.09 \text{ mm}$$

Ques

radius of the circle is 100 mm. Find the area of the shaded region.

Area of the shaded region = Area of the circle - Area of the triangle

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